

Introduction to Real Analysis

Math 452 (Jerzy Kocik)

Topics

0. Preparation

Propositional logic, set theory (Russell paradox), predicative logic, metric spaces, functions, relations, equivalence classes, proofs.

1. What are the Real Numbers

irrationality of $\sqrt{2}$, supremum, the Axiom of Completeness, Archimedean property, cardinality, Cantor's Theorem.

2. Sequences and Series

Rearrangements of infinite Series, algebraic and order limit theorems, monotone convergence theorem, Bolzano-Weierstrass theorem, Cauchy Criterion, properties of infinite series, double summations and products of infinite series.

3. Basic Topology of \mathbf{R}

Set: open, closed, compact, connected, dense.
Heine-Borel Thm. Completion. Baire's Theorem.
Cantor set (+ intro to Hausdorff dimension).

4. Limits and Continuity

Functions (examples: Dirichlet, Thomae, etc), continuity and compactness, Intermediate Value Theorem, Sets of Discontinuity.

5. The Derivative

Intermediate Value Property, Mean Value Thrm, Darboux Thrm, etc.
Examples of a continuous nowhere-differentiable functions.

6. Sequences and Series of Functions:

Uniform convergence of sequences of functions, differentiation, power series (Taylor Series).

7. The Riemann Integral

Definition via Darboux sums, discontinuities, various properties, Fundamental Theorem of Calculus, Lebesgue's criterion for Riemann integrability.

8. Additional Topics

Various constructions of \mathbf{R} from \mathbf{Q} .