Introduction to Real Analysis
Math 452 (Jerzy Kocik)

Topics

0. Preparation
Propositional logic, set theory (Russell paradox), predicative logic, metric spaces, functions, relations, equivalence classes, proofs.

1. What are the Real Numbers
irrationality of $\sqrt{2}$, supremum, the Axiom of Completeness, Archimedean property, cardinality, Cantor's Theorem.

2. Sequences and Series
Rearrangements of infinite Series, algebraic and order limit theorems, monotone convergence theorem, Bolzano-Weierstrass theorem, Cauchy Criterion, properties of infinite series, double summations and products of infinite series.

3. Basic Topology of R

4. Limits and Continuity
Functions (examples: Dirichlet, Thomae, etc), continuity and compactness, Intermediate Value Theorem, Sets of Discontinuity.

5. The Derivative
Intermediate Value Property, Mean Value Thrm, Darboux Thrm, etc. Examples of a continuous nowhere-differentiable functions.

6. Sequences and Series of Functions:
Uniform convergence of sequences of functions, differentiation, power series (Taylor Series).

7. The Riemann Integral
Definition via Darboux sums, discontinuities, various properties, Fundamental Theorem of Calculus, Lebesgue's criterion for Riemann integrability.

8. Additional Topics
Various constructions of $\mathbb{R}$ from $\mathbb{Q}$. 