Cohesive Powers of Galois Extensions

Adjective Salad

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Cohesive Sets

Definition

- Given $A, B \subseteq \mathbb{N}$, we say $A \subseteq^* B$ if $A - B$ is finite.[a]
- A set C is called *cohesive* if for every c.e. set W_e, either $W_e \supseteq^* C$ or $W_e^c \supseteq^* C$.
- For partial computable functions f, g, $f \simeq_C g$ if $\{i \mid f(i)\downarrow = g(i)\downarrow\} \supseteq^* C$.
- This defines an equivalence relation on the set $\{f \mid f \text{ is partial computable and } \mathrm{dom}(f) \supseteq^* C\}$

[a] This is metaphorically like B being measure 1 on A

Cohesive Products

Definition

- Let $\{\mathcal{A}_i\}_{i\in\omega}$ be a uniformly computable sequence of structures in a fixed signature.
- Let $\mathcal{D} = \{\varphi_e \mid \mathrm{dom}(\varphi_e) \supseteq^* C \wedge \forall i(\varphi_e(i)\downarrow \implies \varphi_e(i) \in \mathcal{A}_i)\}$
- As in the ultraproduct, given a relation symbol R in the signature of the $\mathcal{A}_i$'s,

$$\prod_C \mathcal{A}_i \models R([f_1], \ldots, [f_n]) \iff \{i \mid \mathcal{A}_i \models R(f_1(i), \ldots, f_n(i))\} \supseteq^* C$$

- Function and constant symbols are interpreted in the analogous way.
- If the $\mathcal{A}_i$'s are all the same (with a fixed presentation), then $\prod_C \mathcal{A}_i$ is a Cohesive *Power*.

Dimitrov's Theorem for Cohesive Products

Theorem (Dimitrov)

- *If* $\varphi(x_1, \dots, x_n)$ *is a* Σ^0_1 *or* Π^0_1 *formula, then*

$$\prod_C \mathcal{A}_i \models \varphi([f_1], \dots, [f_n]) \iff \{i \mid \mathcal{A}_i \models \varphi(f_1(i), \dots, f_n(i))\} \supseteq^* C$$

- *If* φ *is a* Σ^0_2 *formula*

$$\prod_C \mathcal{A}_i \models \varphi([f_1], \dots, [f_n]) \implies \{i \mid \mathcal{A}_i \models \varphi(f_1(i), \dots, f_n(i))\} \supseteq^* C$$

Dimitrov's Theorem for Cohesive Powers

Theorem (Dimitrov)

- *$\mathcal{A} \hookrightarrow \prod_C \mathcal{A}$ as a two-quantifier elementary substructure.*
- *If $\varphi(x_1, \ldots, x_n)$ is a Σ^0_1 or Π^0_1 formula*

$$\prod_C \mathcal{A} \models \varphi([f_1], \ldots, [f_n]) \iff \{i \mid \mathcal{A} \models \varphi(f_1(i), \ldots, f_n(i))\} \supseteq^* C$$

- *If Ψ is a Σ^0_2 or Π^0_2 sentence*

$$\prod_C \mathcal{A} \models \Psi \iff \mathcal{A} \models \Psi$$

- *If Φ is Σ^0_3*

$$\mathcal{A} \models \Phi \implies \prod_C \mathcal{A} \models \Phi$$

Galois Theory Crash Course

Definition

- A field extension K/F is a pair of fields K/F with $F \subseteq K$.
- A field extension K/F is algebraic if every $\alpha \in K$ is the root of some $p \in F[x]$.
- The degree of a field extension K/F is the dimension of K as an F-vectorspace.
- An algebraic field extension K/F is Galois if it satisfies both of the following:
  - every $p \in F[x]$ that has a root in K, has all of its roots in K (i.e. factors into linear terms in $K[x]$)
  - separable (not important today)
    - Implied by characteristic 0, so imagine extensions of $\mathbb{Q}$.
- If K/F is a Galois extension the group $Gal(K/F) = \{\sigma \in Aut(K) \mid \sigma \restriction F = id\}$ is the Galois group of the extension K/F.

Extensions of Extensions

- Let F, K_1, K_2 be a sequence of fields, with $F \subseteq K_1 \subseteq K_2$, and K_1/F and K_2/F Galois.
- Let $G_i = Gal(K_i/F)$.
- There is a surjective map $G_2 \to G_1$. I.e. $G_1 \cong G_2/H$.
- In fact, $H = Gal(K_2/K_1)$.
- This extends inductively to chains of fields $K_0 \subseteq K_1 \subseteq K_2 \subseteq \dots$

$$F \hookrightarrow K_1 \hookrightarrow K_2 \hookrightarrow K_3 \hookrightarrow \dots$$

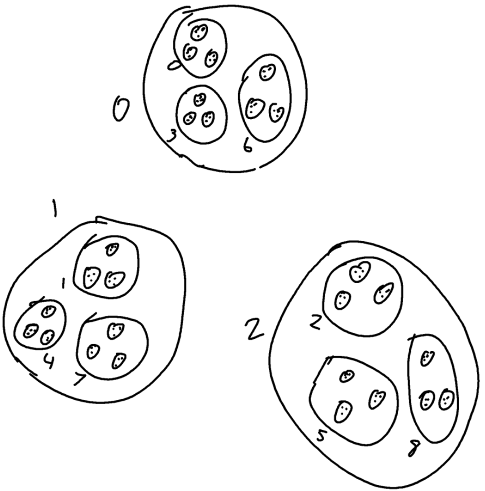
$$1 \longleftarrow G_1 \longleftarrow G_2 \longleftarrow G_3 \longleftarrow \dots$$

Infinte Galois Groups

- Suppose $F \subseteq K_1 \subseteq K_2 \subseteq \ldots$ are fields, with K_i/F fintie Galois, with Galois group G_i, and $K = \bigcup_{i\in\omega} K_i$.
- Then, $Gal(K/F)$ is isomorphic to the projective limit $\varprojlim_{i\in\omega} G_i = \{\{\sigma_i\}_{i\in\omega} \in \prod G_i \mid \sigma_{i+1} \upharpoonright K_i = \sigma_i\} \subseteq \prod G_i$.
- Further, we may place a topology on $Gal(K/F)$, called the profinite topology, where open sets are determined by behaviour on finite sub extensions.

Visualizing Infinite Galois Groups

- Ex: $\mathbb{Z}_3 \cong \varprojlim \mathbb{Z}/3^n\mathbb{Z}$



What we set out to do

K/F

- Goal: Understand $\prod_C K / \prod_C F$.
- Dimitrov conjectured the following:

Conjecture

Let K/F be a computable Galois extension. Then $Gal(\prod_C K / \prod_C F) \cong \prod_C Gal(K/F)$.

- Finite case: Answered positively by Keshav Srinivasan in his thesis (2024).
- Infinite case: problem ill defined!
    - An infinite Galois group is an uncountable topological structure! We can't (or don't know how) to stick it in a cohesive power!

A first theorem

- All the following is joint work with Rumen Dimitrov, Valentina Harizanov, and Keshav Srinivasan.

Theorem

- *Let K/F be a Galois extension. $\prod_C K/\prod_C F$ is algebraic (and further, Galois) if and only if K/F is finite.*
- *Further, if K/F is infinite, and F has a ~~splitting algorithm~~ Factoring, then $\prod_C K/\prod_C F$ has infinite transcendence degree.*

- Can we produce a K/F without a splitting algorithm with $\prod_C K/\prod_C F$ having transcendence degree 1?

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- *Further, if K/F is infinite, and F has a splitting algorithm, then $\prod_C K / \prod_C F$ has infinite transcendence degree.*

- Can we produce a K/F without a splitting algorithm with $\prod_C K / \prod_C F$ having transcendence degree 1?
- It is always the case that $Alg(\prod_C K)/\prod_C F$ is Galois. Let's try instead to understand $Gal(Alg(\prod_C K)/\prod_C F)$.

Psudofinite Extensions

Theorem

- *Suppose* K_i/F *is a sequence of uniformly computable finite Galois extensions, with uniformly computable Galois groups* $G_i \cong Gal(K_i/F)$*, and uniformly computable Galois action* $G_i \curvearrowright K_i$.
- *Then, there is a map* $\prod_C G_i \to Gal(Alg(\prod_C K_i)/\prod_C F)$.
- *The image of this map is dense in the profinite topology on* $Gal(Alg(\prod_C K_i)/\prod_C F)$
- *The kernel of this map is exactly* $\{[\sigma] : \forall n, \{x : \sigma(x) \in \bigcap_{H,[H:G]=n} H\} \supseteq^* C\}\}$.

Sequence Extension Lemma

Definition

Let A_i be a uniformly computable sequence of structures, and let $\varphi : \mathbb{N} \to \mathbb{N}$ be a partial computable function with $\mathrm{dom}(\varphi) \supseteq^* C$.

$$\prod_{C,\varphi} A_i := \prod_{C} A_{\varphi(i)}$$

Proposition

Let φ_1, φ_2 be partial computable functions defined almost everywhere. If $\varphi_1 \simeq_C \varphi_2$, then $\prod_{C,\varphi_1} A_i \cong \prod_{C,\varphi_2} A_i$.

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Observation

The conditions that $\mathrm{dom}(\varphi_1), \mathrm{dom}(\varphi_2) \supseteq^* C$ and $\varphi_1 \simeq_C \varphi_2$ is exactly the same as saying $[\varphi_1] = [\varphi_2] = M \in \prod_C \mathbb{N}$!
Thus, if $[\varphi] = M \in \prod_C \mathbb{N}$, we will write $\prod_{C,M} A_i = \prod_C A_{\varphi(i)}$

Sequence Extension Lemma

Lemma

- *Let A_i be a uniformly computable sequence of structures. Then, we may extend the sequence* $\prod_C A_0, \prod_C A_1, \ldots$ *to* $\{\prod_{C,M} A_i\}_{M \in \mathbb{N}}$.

Sequence Extension Lemma

Lemma

- *Let A_i be a uniformly computable sequence of structures. Then, we may extend the sequence $\prod_C A_0, \prod_C A_1, \dots$ to $\{\prod_{C,M} A_i\}_{M \in \mathbb{N}}$.*
- *Further, if there are uniformly computable maps $A_i \to A_{i+1}$, then there are natural maps $\prod_{C,M} A_i \to \prod_{C,M+1} A_i$.*
  - *In particular, if $A_i \hookrightarrow A_{i+1}$, then $\prod_{C,M} A_i \hookrightarrow \prod_{C,M+1} A_i$.*

Sequence Extension Lemma

Lemma

- *Let A_i be a uniformly computable sequence of structures. Then, we may extend the sequence* $\prod_C A_0, \prod_C A_1, \ldots$ *to* $\{\prod_{C,M} A_i\}_{M \in \mathbb{N}}$.
- *Further, if there are uniformly computable maps* $A_i \to A_{i+1}$*, then there are natural maps* $\prod_{C,M} A_i \to \prod_{C,M+1} A_i$.
  - *In particular, if* $A_i \hookrightarrow A_{i+1}$*, then* $\prod_{C,M} A_i \hookrightarrow \prod_{C,M+1} A_i$.
- *Further still, if* $\bigcup_{i \in \mathbb{N}} A_i = B$*, then* $\bigcup_{I \in \prod_C \mathbb{N}} \prod_{C,I} A_i = \prod_C B$

Fully Computable Galois Extensions

Definition

A computable infinite Galois extension K/F is fully computable if:

- There are uniformly computable subfields K_i with each K_i/F finite Galois.
- $K_i \hookrightarrow K_{i+1}$ and $K_i \hookrightarrow K$ uniformly computably.
- $G_i \cong Gal(K_i/F)$ are uniformly computable, with $G_{i+1} \to G_i$ uniformly computable, and $G_i \curvearrowright K_i$ uniformly computable.
- These fields are quite abundant.
- e.g. $\overline{\mathbb{Q}}/\mathbb{Q}$, $\mathbb{Q}^{ab}/\mathbb{Q}$.
- Also $\mathbb{F}_p(t)^{sep}/\mathbb{F}_p$.

Density in the colimit

Theorem

Let K/F be a fully computable infinite Galois extension. Then, there is a map $\varprojlim_{M \in \prod_C \mathbb{N}} \prod_{C,M} G_i \to Gal(Alg(\prod_C K)/\prod_C F)$. Further, the kernel and image are controlled by the projective limit.

Density in the colimit: diagram form

$$
\begin{array}{ccccccc}
0 & \longrightarrow & KerF & \hookrightarrow & \varprojlim_{M \in \prod_C \mathbb{N}} \prod_{C,M} G_i & \longrightarrow & Gal(Alg(\prod_C K)/\prod_C F) \\
 & & \downarrow & & \downarrow & & \downarrow \\
 & & \vdots & & \vdots & & \vdots \\
 & & & & & & \\
0 & \longrightarrow & KerF_{M_1} & \hookrightarrow & \prod_{C,M_1} G_1 & \xrightarrow{F_{M_1}} & Gal\left(Alg\left(\prod_{C,M_1} K_i\right)/(\prod_C F)\right) \\
 & & \downarrow & & \downarrow & & \downarrow \\
0 & \longrightarrow & KerF_{M_2} & \hookrightarrow & \prod_{C,M_2} G_i & \xrightarrow{F_{M_2}} & Gal\left(Alg\left(\prod_{C,M_2} K_i\right)/(\prod_C F)\right) \\
 & & \downarrow & & \downarrow & & \downarrow \\
 & & \vdots & & \vdots & & \vdots
\end{array}
$$

What is a transcendental, really?

- Let $F = \mathbb{Q}$.
- Let $K_i = \mathbb{Q}(\sqrt[i]{2})$.
- Consider $f(i) = (\sqrt[i]{2})$.
- $[f] \in \prod_C K_i$, and is transcendental over $\prod_C \mathbb{Q}$.

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- Consider $f(i) = (\sqrt[i]{2})$.
- $[f] \in \prod_C K_i$, and is transcendental over $\prod_C \mathbb{Q}$.
- But it looks like $\sqrt[\omega]{2}$ to me!

Hyper-polynomials

Definition

- Let R be a computable ring (e.g. $\mathbb{Z}, \mathbb{Q}, K$). A hyper-polynomial is an element of the cohesive power $\prod_C R[x_0, x_1, \ldots]$.

Idea

Anything computable transfers to the cohesive power.

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  - $deg([p] + [q]) \leq \max\{\deg([p]), deg([q])\}$ and $\deg([p][q]) = deg([p]) + deg([q])$.

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  - $deg([p] + [q]) \leq \max\{\deg([p]), deg([q])\}$ and $\deg([p][q]) = deg([p]) + deg([q])$.
- Elements of $\prod_C \mathbb{Z}[x_1, x_2, \ldots]$ have a "count" of variables $[f] \in \prod_C \mathbb{N}$.

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Anything computable transfers to the cohesive power.

- Elements of $\prod_C \mathbb{Z}[x]$ have well defined *degree*. ($deg : \prod_C \mathbb{Z}[x] \to \prod_C \mathbb{N}$)
  - $deg([p] + [q]) \leq \max\{\deg([p]), deg([q])\}$ and $\deg([p][q]) = deg([p]) + deg([q])$.
- Elements of $\prod_C \mathbb{Z}[x_1, x_2, \ldots]$ have a "count" of variables $[f] \in \prod_C \mathbb{N}$.
- Most most importantly, we can *evaluate* hyper polynomials.

Evaluating a Hyper-polynomial

- n-variate polynomials take in an n-tuple as an argument. So to evaluate a hyper-polynomial with $M \in \prod_C \mathbb{N}$ variables, we need a non-standard M-tuple of elements from $\prod_C K_i$.
- M-tuples live in $\prod_C K_i^{<\omega}$.

Lemma

- *There is a map* $|\cdot| : \prod_C K^{<\omega} \to \prod_C \mathbb{N}$ *that "counts the length of tuples* $M \in \prod_C K^{<\omega}$*".*
  - *One can show that elements of* $\prod_C K_i^{<\omega}$ *are "just" lists of elements of* $\prod_C K_i$*, indexed over initial segments of* $\prod_C \mathbb{N}$*.*
- *There is a map* $Eval : (\prod_C \mathbb{Z}[x_1 \ldots]) \times \prod_C K^{<\omega}$*, that, when* $p \in \prod_C \mathbb{Z}[x_1, x_2, \ldots]$ *is* M*-variate, and* $\vec{a} \in \prod_C K_i^{<\omega}$ *is an* M*-touple,* $Eval(p, a)$ *is an element of* K*.* $\prod_C K_i$
- *Moreover,* $Eval$ *"behaves as it should"*
  - $Eval$ *is a ring homomorphism in the first coordinate.*
  - *When* p *is a standard polynomial,* $Eval$ *agrees with the actual value* $p(a_1, \ldots, a_M)$*.*

An Example of Hyper-polynomial Evaluation

- Let $p(i) = x_1 x_2 \dots x_i$.
- Since p is total computable, and the range of p is contained in $\mathbb{Z}[x_1, x_2, \dots]$, we have $[p]$ is an element of $\prod_C \mathbb{Z}[x_1, x_2, \dots]$.
- We should think of $[p]$ as being the hyper-polynomial $\prod_{J \leq [id]} x_J$, where J ranges over elements of $\prod_C \mathbb{N}$.
- $[p]$ has $[id]$ many variables.
- Let $\vec{a}(i) = (1, 2, \dots, i)$. Similar to above, $[\vec{a}] \in \prod_C \mathbb{N}^{<\omega}$, and is an $[id]$-tuple.
- We evaluate $[p]([\vec{a}])$ by setting it equal to $[b]$, where $b(i) = p(i)((\vec{a}(i)) = \prod_{j \leq i} j = i!$, that is $[id]$ factorial.
- Thus we may think of $[p]([\vec{a}]) = [id]!$.

Hyper-automorphisms

The Goal of the hyper-automorphism group is to capture the automorphisms of the field extension $\prod_C K_i / \prod_C F$ that preserve all of the hyper-polynomial structure.

Definition

- Let K_i/F be a uniformly computable sequence of field extensions.
- Thinking of $\prod_C K_i^{<\omega}$ as a cohesive products of trees, let

$$HAut\left(\prod_C K_i^{<\omega}\right) = \left\{\sigma \in Aut\left(\prod_C K_i^{<\omega}\right) \middle| (\forall \vec{a} \in \prod_C K_i^{<\omega})(|\vec{a}| = |\sigma \vec{a}|)\right\}$$

- Now let $HAut(\prod_C K_i)$ be the set of all $\sigma \in HAut(\prod_C K_i^{<\omega})$ that satisfy

$$\forall p \in \prod_C (\mathbb{Z}[x_1, x_2, \dots]), \forall \vec{a} \in \prod_C K_i^{<\omega}, [|\vec{a}| = deg(p) \implies p(\sigma(\vec{a})) = \sigma(p(\vec{a}))]$$

- Further, let

$$HAut(\textstyle\prod_C K_i / \prod_C F) = \{\sigma \in HAut\left(\prod_C K_i\right) | \sigma \restriction \prod_C F^{<\omega} = id\}$$

Hyper-automorphism Group of Psudofinite Galois Extensions

Theorem

Let K_i/F be a uniformly computable sequence of computable finite Galois extensions, with uniformly computable Galois groups and actions $G_i \curvearrowright K_i$. Then

$$HAut(\prod_C K_i/\prod_C F) \cong \prod_C G_i$$

- Key idea: psudofinite extensions behave like finite extensions with respect to hyperpolynomials.
- There is a single element $\alpha \in \prod_C K_i$ that governs $HAut(\prod_C K_i/\prod_C F)$.

Hyper-automorhpism Group of infinite Galois Extensions

Theorem

Let K/F with $K_1 \subseteq K_2 \subseteq \ldots$, and $G_i \cong Gal(K_i/F)$ be a fully computable Galois extension. Then,

$$HAut(\prod_C K_i / \prod_C F) \cong \varprojlim_{M \in \prod_C \mathbb{N}} \prod_{C,M} G_i$$

Proof.

$$HAut(\prod_C K_i / \prod_C F) \cong \varprojlim_{M \in \prod_C \mathbb{N}} HAut(\prod_{C,M} K_i / \prod_C F) \cong \varprojlim_{M \in \prod_C \mathbb{N}} \prod_{C,M} G_i$$

□

Metric On Infinite Galois Extensions

Definition

- Let K/F be an infinite (countable) Galois extension, with sub-extensions $K_0 \subseteq K_1 \subseteq K_2 \subseteq \cdots \subseteq K$, with each K_i/F finite.
- Let $G = Gal(K/F)$ be the infinite Galois group, and let $\sigma, \tau \in G$.
- Set $d(\sigma,\tau) = \begin{cases} 0 & \sigma = \tau \\ \frac{1}{2^n} & \sigma \neq \tau \text{ and } n = \min\{m \mid \sigma \restriction K_m \neq \tau \restriction K_m\} \end{cases}$

- The metric carries more information about the decomposition of K into the K_i's than the algebra of K.

Hyper-metric Type Structure of Hyper-automorphism Groups of Cohesive Powers of Infinite Galois Extension

Definition

Let $\sigma, \tau \in HAut(\prod_C K / \prod_C F)$, where K/F is a fully computable Galois extension. Set

$$d(\sigma,\tau) = \begin{cases} 0 & \text{if } \sigma = \tau; \\ \frac{1}{2^N} & \text{where } N \text{ is the least element of } \prod_C \mathbb{N} \text{ such that } \sigma \restriction \prod_{C,N} K_i \neq \tau \restriction \prod_{C,N} K_i, \end{cases}$$

where $\frac{1}{2^N}$ is understood to be an element of $\prod_C \mathbb{Q}$.

- Somewhat surprisingly, a least such N always exists, despite $\prod_C \mathbb{N}$ having almost no induction.

Modulo the Infinitesimals

- Now, consider $\sigma \in HAut(\prod_C K / \prod_C F)$.
- Call σ an infinitesimal if $d(\sigma, id)$ is 0 or non-standard. $\frac{1}{2^n}$
- The set of infinitesimals I is a normal subgroup of $HAut(\prod_C K / \prod_C F)$.

Theorem

$HAut(\prod_C K / \prod_C F)/I$ is isometrically isomorphic to a dense subgroup of $Gal(K/F)$.

The Map $\prod_C G_i \to Gal(Alg(\prod_C K)/\prod_C F)$

Definition

Let $\{G_i\}_{i\in\omega}$ be a uniformly computable sequence of finite groups, and let $\{H_i\}_{i\in\omega}$ be a uniformly computable sequence of subgroups $H_i \leq G_i$. We call the group $\prod_C H_i$ an internal subgroup of $\prod_C G_i$.

- Recall that $HAut(\prod_C K_i/\prod_C F)$ is isomorphic to $\prod_C G_i$

Theorem

The map $HAut(\prod_C K_i/\prod_C F) \to Gal(Alg(\prod_C K_i)/\prod_C F)$ has kernel exactly the intersection of all internal subgroups of $\prod_C G_i$ of finite index.

The Map $HAut(\prod_C K / \prod_C F) \to Gal(Alg(\prod_C K)/\prod_C F)$

Theorem

Let K/F, $\{K_i\}_{i\in\omega}$, $\{G_i\}_{i\in\omega}$ be a fully computable Galois extension, and let $N, M \in \prod_C \mathbb{N}$ with $N < M$. Then, the following diagram commutes:

$$
\begin{array}{ccccc}
ker\Psi & \longrightarrow & HAut(\prod_C K/\prod_C F) & \xrightarrow{\Psi} & Gal(Alg(\prod_C K)/\prod_C F) \\
\Vert & & \Vert & & \Vert \\
\varprojlim \bigcap_{\{H_{N(i)}\}} \prod_C H_{N(i)} & \longrightarrow & \varprojlim \prod_{C,N} G_i & \longrightarrow & \varprojlim Gal(Alg(\prod_{C,N} K_i)/\prod_C F) \\
\downarrow & & \downarrow & & \downarrow \\
\bigcap_{\{H_{M(i)}\}} \prod_C H_{M(i)} & \longrightarrow & \prod_{C,M} G_i \cong HAut(\prod_{C,M} K_i/\prod_C F) & \longrightarrow & Gal(Alg(\prod_{C,M} K_i)/\prod_C F) \\
\downarrow & & \downarrow & & \downarrow \\
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\end{array}
$$

Thank You!

Rumen Dimitrov et al. *On Cohesive Products of Fields*. 2026. arXiv: 2604.09965 [math.LO]. URL: https://arxiv.org/abs/2604.09965