1. We let x represent A's wages, y represent B's wages, and z represent C's wages, which we are told equal \$30,000. The amount paid out by each A, B, and C equals the amount each receives, so

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ giving the system } \begin{cases} x = \frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z \\ y = \frac{1}{4}x + \frac{1}{3}y + \frac{1}{4}z \\ z = \frac{1}{4}x + \frac{1}{3}y + \frac{1}{2}z \end{cases} \text{ or } \begin{cases} \frac{1}{2}x - \frac{1}{3}y - \frac{1}{4}z = 0 \\ -\frac{1}{4}x + \frac{2}{3}y - \frac{1}{4}z = 0 \\ -\frac{1}{4}x - \frac{1}{3}y + \frac{1}{2}z = 0 \end{cases}$$

Solving the equations for x, y, and z, we find

$$\left[\begin{array}{ccc|c} \frac{1}{2} & -\frac{1}{3} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & \frac{2}{3} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & -\frac{1}{3} & \frac{1}{2} & 0 \end{array}\right] \xrightarrow[\substack{R_1 = 2r_1 \\ R_2 = \frac{1}{4}r_1 + r_2 \\ R_3 = \frac{1}{4}r_1 + r_3}]{} \left[\begin{array}{ccc|c} 1 & -\frac{2}{3} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{3}{8} & 0 \\ 0 & -\frac{1}{2} & \frac{3}{8} & 0 \end{array}\right] \xrightarrow[\substack{R_2 = 2r_2 \\ R_1 = \frac{2}{3}r_2 + r_1 \\ R_3 = \frac{1}{2}r_2 + r_3}]{} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

or $x = z$ and $y = \frac{3}{4}z$ where z is the parameter. Since we are told C's wages are \$30,000, we know A's wages are also \$30,000, and B's wages are $\frac{3}{4}(30{,}000) = \$22{,}500$.

3. We let x represent A's wages, y represent B's wages, and z represent C's wages, which we are told equal \$30,000. Since the amount paid out by each A, B, and C equals the amount each receives, we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.2 & 0.3 & 0.1 \\ 0.6 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ which gives the system}$$

$$\begin{cases} x = 0.2x + 0.3y + 0.1z \\ y = 0.6x + 0.4y + 0.2z \\ z = 0.2x + 0.3y + 0.7z \end{cases} \text{ or } \begin{cases} 0.8x - 0.3y - 0.1z = 0 \\ -0.6x + 0.6y - 0.2z = 0 \\ -0.2x - 0.3y + 0.3z = 0 \end{cases} \text{ or } \begin{cases} 8x - 3y - 1z = 0 \\ -6x + 6y - 2z = 0 \\ -2x - 3y + 3z = 0 \end{cases}$$

Solving the equations for x, y, and z, we find

$$\left[\begin{array}{ccc|c} 8 & -3 & -1 & 0 \\ -6 & 6 & -2 & 0 \\ -2 & -3 & 3 & 0 \end{array}\right] \xrightarrow[\substack{R_1 = \frac{1}{8}r_1 \\ R_2 = 6r_1 + r_2 \\ R_3 = 2r_1 + r_3}]{} \left[\begin{array}{ccc|c} 1 & -\frac{3}{8} & -\frac{1}{8} & 0 \\ 0 & \frac{15}{4} & -\frac{11}{4} & 0 \\ 0 & -\frac{15}{4} & \frac{11}{4} & 0 \end{array}\right] \xrightarrow[\substack{R_2 = \frac{4}{15}r_2 \\ R_1 = \frac{3}{8}r_2 + r_1 \\ R_3 = \frac{15}{4}r_2 + r_3}]{} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{2}{5} & 0 \\ 0 & 1 & -\frac{11}{15} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

or $x = \frac{2}{5}z$ and $y = \frac{11}{15}z$ where z is the parameter. Since we are told C's wages are \$30,000, we find A's wages are $\frac{2}{5}(30{,}000) = \$12{,}000$, and B's wages are $\frac{11}{15}(30{,}000) = \$22{,}000$.

5. The total output vector X for an open Leontief model is from the system $X = [I_3 - A]^{-1} \cdot D$ where D represents future demand for the goods produced in the system. If $D_2 = \begin{bmatrix} 80 \\ 90 \\ 60 \end{bmatrix}$, then using the information from Table 5, we get

$$X = \begin{bmatrix} 1.6048 & 0.3568 & 0.7131 \\ 0.2946 & 1.3363 & 0.3857 \\ 0.3660 & 0.2721 & 1.4013 \end{bmatrix} \begin{bmatrix} 80 \\ 90 \\ 60 \end{bmatrix} = \begin{bmatrix} 203.28 \\ 166.98 \\ 137.85 \end{bmatrix}$$

The total output of R, S, and T required for the forecast demand D_2 is to produce 203.28 units of product R, 166.98 units of product S, and 137.85 units of product T.

6. The total output vector X for an open Leontief model is from the system $X = [I_3 - A]^{-1} \cdot D$ where D represents future demand for the goods produced in the system. If $D_4 = \begin{bmatrix} 100 \\ 80 \\ 60 \end{bmatrix}$, then using the information from Table 5, we get

$$X = \begin{bmatrix} 1.6048 & 0.3568 & 0.7131 \\ 0.2946 & 1.3363 & 0.3857 \\ 0.3660 & 0.2721 & 1.4013 \end{bmatrix} \begin{bmatrix} 100 \\ 80 \\ 60 \end{bmatrix} = \begin{bmatrix} 231.81 \\ 159.51 \\ 142.45 \end{bmatrix}$$

The total output of R, S, and T required for the forecast demand D_4 is to produce 231.81 units of product R, 159.51 units of product S, and 142.45 units of product T.