11. $\left[\begin{array}{ccc|c} 2 & -3 & 1 & 7 \\ 1 & 1 & -1 & 1 \\ 2 & 2 & -3 & -4 \end{array}\right]$

12. $\left[\begin{array}{ccc|c} 5 & -3 & 6 & -1 \\ -1 & -1 & 1 & 1 \\ 2 & 3 & 0 & -5 \end{array}\right]$

18. $$\left[\begin{array}{cc|c} 1 & -3 & -3 \\ 2 & -5 & -4 \end{array}\right] \xrightarrow{R_2=-2r_1+r_2} \left[\begin{array}{cc|c} 1 & -3 & -3 \\ -2(1)+2 & -2(-3)+(-5) & -2(-3)+(-4) \end{array}\right] = \left[\begin{array}{cc|c} 1 & -3 & -3 \\ 0 & 1 & 2 \end{array}\right]$$

19. (a) $$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 2 & -5 & 6 & 6 \\ -3 & 3 & 4 & 6 \end{array}\right] \xrightarrow{R_2=-2r_1+r_2} \left[\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ -2(1)+2 & -2(-3)-5 & -2(4)+6 & -2(3)+6 \\ -3 & 3 & 4 & 6 \end{array}\right]$$
$$= \left[\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 0 & 1 & -2 & 0 \\ -3 & 3 & 4 & 6 \end{array}\right]$$

(b) $$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 2 & -5 & 6 & 6 \\ -3 & 3 & 4 & 6 \end{array}\right] \xrightarrow{R_3=3r_1+r_3} \left[\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 2 & -5 & 6 & 6 \\ 3(1)-3 & 3(-3)+3 & 3(4)+4 & 3(3)+6 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 2 & -5 & 6 & 6 \\ 0 & -6 & 16 & 15 \end{array}\right]$$

26. (a) $$\begin{cases} x-3y=-4 \\ \quad\;\; y=\;\;0 \end{cases}$$

(b) The system is consistent.
The solution is $x = -4$, $y = 0$ or $(-4, 0)$.

29. (a) $$\begin{cases} x+2z=-1 \\ y-4z=-2 \\ \qquad 0=\;\;0 \end{cases}$$

(b) The system is consistent and has an infinite number of solutions. The solutions are:
$x = -2z - 1$, $y = 4z - 2$ where z is any real number.

32. (a) $$\begin{cases} x_1+2x_2+4x_3=\;1 \\ \quad x_2-x_3+2x_4=\;2 \\ \qquad\quad x_3+3x_4=\;0 \\ \qquad\qquad\quad x_4=-2 \end{cases}$$

(b) The system is consistent.
To find the solution start with $x_4 = -2$ and back-substitute.

$x_3 = -3x_4 = -3(-2) = 6$
$x_2 = -2x_4 + x_3 + 2 = -2(-2) + 6 + 2 = 12$
$x_1 = -4x_3 - 2x_2 + 1 = -4(6) - 2(12) + 1 = -47$

or $(-47, 12, 6, -2)$

40. Write the system as

$$\left[\begin{array}{cc|c} 3 & 2 & 7 \\ 1 & 1 & 3 \end{array}\right] \xrightarrow[\text{rows 1 and 2}]{\text{Interchange}} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 3 & 2 & 7 \end{array}\right] \xrightarrow{R_2 = -3r_1 + r_2} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -1 & -2 \end{array}\right] \xrightarrow{R_2 = -r_2} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 2 \end{array}\right]$$

The row-echelon form of the system is $\begin{cases} x + y = 3 \\ \quad\; y = 2 \end{cases}$

Back-substitute 2 for y in the first equation giving $x + 2 = 3$, or $x = 1$.
The solution of the system of equations is $x = 1$ and $y = 2$ or (1, 2).

43. Write the system as

$$\left[\begin{array}{cc|c} 2 & -3 & 6 \\ 6 & -9 & 10 \end{array}\right] \xrightarrow{R_1 = \frac{1}{2}r_1} \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 3 \\ 6 & -9 & 10 \end{array}\right] \xrightarrow{R_2 = -6r_1 + r_2} \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 3 \\ 0 & 0 & -8 \end{array}\right]$$

The system is inconsistent.

47. Write the system as

$$\left[\begin{array}{cc|c} 2 & 6 & 4 \\ 5 & 15 & 10 \end{array}\right] \xrightarrow{R_1 = \frac{1}{2}r_1} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 5 & 15 & 10 \end{array}\right] \xrightarrow{R_2 = -5r_1 + r_2} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

This system has an infinite number of solutions. They are $x = 2 - 3y$, where y is any real number, or written as ordered pairs $\{(x, y) | x = -3y + 2,\ y \text{ any real number}\}$.

53. Write the system as

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 6 \\ 1 & -1 & -1 & -3 \\ 3 & 1 & 2 & 7 \end{array}\right] \xrightarrow[\text{rows 1 and 2}]{\text{Interchange}} \left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 2 & 1 & 1 & 6 \\ 3 & 1 & 2 & 7 \end{array}\right] \xrightarrow[R_3 = -3r_1 + r_3]{R_2 = -2r_1 + r_2} \left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 3 & 3 & 12 \\ 0 & 4 & 5 & 16 \end{array}\right]$$

$$\xrightarrow{R_2 = \frac{1}{3}r_2} \left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & 1 & 4 \\ 0 & 4 & 5 & 16 \end{array}\right] \xrightarrow{R_3 = -4r_2 + r_3} \left[\begin{array}{ccc|c} 1 & -1 & -1 & -3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

The row echelon form of the system of equations is

$$\begin{cases} x = y + z - 3 & (1) \\ y = -z + 4 & (2) \\ z = 0 & (3) \end{cases}$$

Back-substitute 0 for z in (2) to get $y = 4$.
Then back-substitute $z = 0$ and $y = 4$ in equation (1), to get $x = 4 + 0 - 3 = 1$.

The solution of the system of equations is $x = 1$, $y = 4$, and $z = 0$ or written as an ordered triple, (1, 4, 0).

56. Write the system as

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & -5 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 5 \end{array}\right] \xrightarrow[\text{rows 1 and 2}]{\text{Interchange}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & -1 & -1 & -5 \\ 1 & 2 & 2 & 5 \end{array}\right] \xrightarrow[\substack{R_2 = -2r_1 + r_2 \\ R_3 = -r_1 + r_3}]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -3 & -3 & -9 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$\xrightarrow[\text{rows 2 and 3}]{\text{Interchange}} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & -3 & -3 & -9 \end{array}\right] \xrightarrow[R_3 = 3r_2 + r_3]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The row echelon form of the system of equations is

$$\begin{cases} x = -y - z + 2 & (1) \\ y = -z + 3 & (2) \end{cases}$$

This system has an infinite number of solutions. Back-substitute $y = -z + 3$ in equation (1), to get $x = -(-z + 3) - z + 2 = z - 3 - z + 2 = -1$.

The solution of the system of equations is $x = -1$ and $y = -z + 3$, where z is any real number.