8. To be a solution to the system of equations, the given values of the variables must solve each equation. So we evaluate each equation at $x = 2$ and $y = 4$.

$$\begin{aligned} 3x + 2y &= 2 \\ 3(2) + 2(4) &= 14 \neq 2 \end{aligned}$$

Since the first equation was not satisfied, $x = 2$, $y = 4$ is not a solution to the system.

9. To be a solution to the system of equations, the given values of the variables must solve each equation. So we evaluate each equation at $x = 2$ and $y = \frac{1}{2}$.

$$\begin{aligned} 3x + 4y &= 4 \\ 3(2) + 4\left(\frac{1}{2}\right) &= 8 \neq 4 \end{aligned}$$

Since the first equation is not satisfied, $x = 2$, $y = \frac{1}{2}$ is not a solution to the system.

15. To be a solution to the system of equations, the given values of the variables must solve each equation. So we evaluate each equation at $x = 2$, $y = -2$, and $z = 2$.

$$\begin{aligned} 3x + 3y + 2z &= 4 \\ 3(2) + 3(-2) + 2(2) &= 4 \end{aligned} \qquad \begin{aligned} x - 3y + z &= 10 \\ (2) - 3(-2) + 2 &= 10 \end{aligned} \qquad \begin{aligned} 5x - 2y - 3z &= 8 \\ 5(2) - 2(-2) - 3(2) &= 8 \end{aligned}$$

Since all three equations are satisfied, $x = 2$, $y = -2$, $z = 2$ is a solution to the system.

18. Choosing the method of elimination to solve this system,

$$\begin{cases} x + 2y = 5 & (1) \\ x + y = 3 & (2) \end{cases}$$

$$\begin{aligned} 2x + 2y &= 6 \quad (2) && \text{Multiply (2) by 2.} \\ x &= 1 && \text{Subtract (1) from (2).} \\ 1 + y &= 3 && \text{Back-substitute 1 for } x \text{ in equation (2).} \\ y &= 2 && \text{Solve for } y. \end{aligned}$$

The solution to the system is $x = 1$, $y = 2$, or written as an ordered pair, $(1, 2)$.

24. Choosing the method of elimination to solve this system,

$$\begin{cases} 2x + 4y = \dfrac{2}{3} & (1) \\ 3x - 5y = -10 & (2) \end{cases}$$

$6x + 12y = 2 \quad (1)$	Multiply equation (1) by 3.
$6x - 10y = -20 \quad (2)$	Multiply equation (2) by 2.
$22y = 22$	Subtract (2) from (1).
$y = 1$	Solve for y.
$3x - 5(1) = -10$	Back-substitute 1 for y in equation (2).
$x = -\dfrac{5}{3}$	Solve for x.

The solution to the system is $x = -\dfrac{5}{3}$, $y = 1$, or written as an ordered pair, $\left(-\dfrac{5}{3}, 1\right)$.

36. Choosing the method of elimination to solve this system,

$$\begin{cases} \dfrac{1}{3}x - \dfrac{3}{2}y = -5 & (1) \\ \dfrac{3}{4}x + \dfrac{1}{3}y = 11 & (2) \end{cases}$$

$2x - 9y = -30 \quad (1)$	Multiply equation (1) by 6 to remove the fractions.
$9x + 4y = 132 \quad (2)$	Multiply equation (2) by 12 to remove the fractions.
$8x - 36y = -120 \quad (1)$	Multiply equation (1) by 4.
$81x + 36y = 1188 \quad (2)$	Multiply equation (2) by 9.
$89x = 1068$	Add equations (1) and (2).
$x = 12$	Solve for x.
$2(12) - 9y = -30 \quad (1)$	Back-substitute 12 for x in equation (1).
$-9 = -54$	Simplify.
$y = 6$	Solve for y.

The solution of the system is $x = 12$, $y = 6$, or written as an ordered pair, (12, 6).

41.

$$\begin{cases} x-2y+3z=7 & (1) \\ 2x+y+z=4 & (2) \\ -3x+2y-2z=-10 & (3) \end{cases}$$

$$\begin{array}{rl} x-2y+3z=7 & (1) \\ 2x+y+z=4 & (2) \end{array} \quad \text{Multiply by 2} \quad \begin{array}{rl} x-2y+3z=7 & (1) \\ \underline{4x+2y+2z=8} & (2) \\ 5x\qquad +5z=15 & \text{Add} \end{array}$$

$$\begin{array}{rl} x-2y+3z=7 & (1) \\ \underline{-3x+2y-2z=-10} & (2) \\ -2x+\qquad z=-3 & \text{Add} \end{array}$$

$$\begin{cases} x-2y+3z=7 & (1) \\ 5x+\qquad 5z=15 & (2) \\ -2x+\qquad z=-3 & (3) \end{cases}$$

Working only with equations (2) and (3),

$$\begin{array}{rl} 5x+5z=15 & (2) \\ -2x+z=-3 & (3) \end{array} \quad \text{Multiply by } (-5) \quad \begin{array}{rl} 5x+5z=15 & (2) \\ \underline{10x-5z=15} & (3) \\ 15x\qquad =30 & \text{or} \quad x=2 \end{array}$$

43.

$$\begin{cases} x-y-z=1 & (1) \\ 2x+3y+z=2 & (2) \\ 3x+2y=0 & (3) \end{cases}$$

Since (3) has no z term we will eliminate z first.

$$\begin{array}{rl} x-y-z=1 & (1) \\ \underline{2x+3y+z=2} & (2) \\ 3x+2y\qquad =3 & \text{(Add) (2)} \end{array}$$

We now use the revised (2) and (3).

$$\begin{array}{rl} 3x+2y=3 & (2) \\ \underline{3x+2y=0} & (3) \\ 0=3 & \text{(Subtract) (3)} \end{array}$$

Equation (3) has no solution; the system is inconsistent.

45. $$\begin{cases} x - y - z = 1 & (1) \\ -x + 2y - 3z = -4 & (2) \\ 3x - 2y - 7z = 0 & (3) \end{cases}$$

Eliminate x:

$$\begin{array}{rl} x - y - z = 1 & (1) \\ -x + 2y - 3z = -4 & (2) \\ \hline y - 4z = -3 & \text{(Add) } (2) \end{array}$$

$$\begin{array}{rl} x - y - z = 1 & (1) \\ 3x - 2y - 7z = 0 & (3) \end{array} \quad \text{Multiply by } -3 \quad \begin{array}{rl} -3x + 3y + 3z = -3 & (1) \\ 3x - 2y - 7z = 0 & (3) \\ \hline y - 4z = -3 & \text{(Add) } (3) \end{array}$$

We now used revised (2) and (3).

$$\begin{array}{rl} y - 4z = -3 & (2) \\ y - 4z = -3 & (3) \\ \hline 0 = 0 & (3) \end{array}$$

The original system is equivalent to a system containing 2 equations, so the equations are dependent and the system has infinitely many solutions. If z represents any real number, then substituting $y = 4z - 3$ into (1) gives

$$x - y - z = 1 \quad (1)$$

$$x - (4z - 3) - z = 1 \text{ or } x = 5z - 2$$

The solution to the system is $\begin{cases} x = 5z - 2 \\ y = 4z - 3 \end{cases}$ where z is any real number.